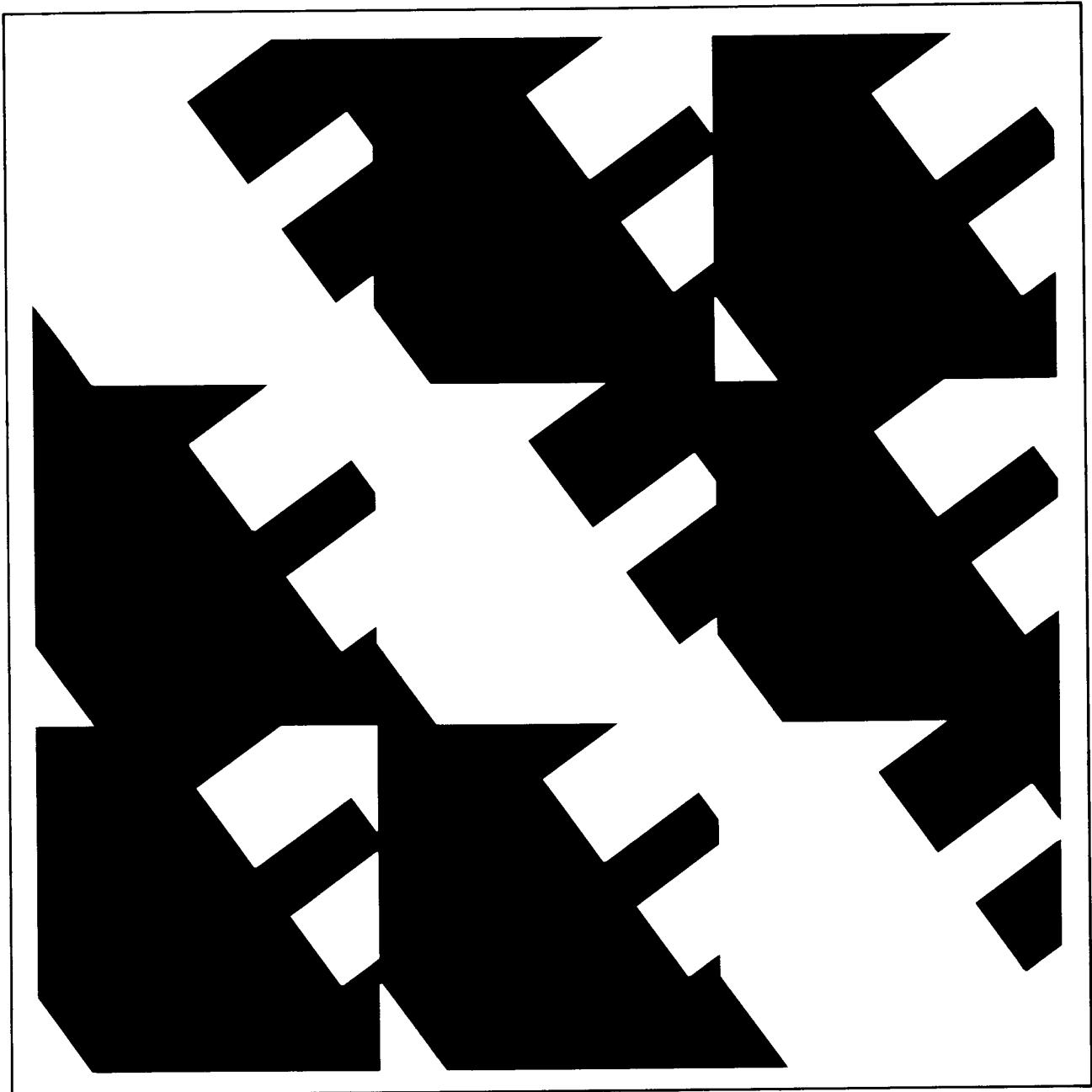


IEEE Recommended Practice: Definitions of Basic Per-Unit Quantities for AC Rotating Machines



IEEE Std 86-1987



Published by The Institute of Electrical and Electronics Engineers, Inc 345 East 47th Street, New York, NY 10017, USA

February 16, 1987

SH10900

**IEEE Recommended Practice:
Definitions of Basic Per-Unit Quantities for
AC Rotating Machines**

Sponsor

**Rotating Machinery Committee of the
IEEE Power Engineering Society**

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Foreword

(This Foreword is not a part of IEEE Std 86-1987, IEEE Recommended Practice: Definitions of Basic Per-Unit Quantities for AC Rotating Machines.)

When machine parameters are expressed in a properly selected per-unit system, the parameters of a wide range of machine sizes fall into relatively narrow ranges even though the actual values of these same parameters may vary over extremely wide ranges. The per-unit system thereby makes possible an easy comparison between different machines, essentially independent of size, without the use of conversion factors, and facilitates the identification of gross calculation errors.

The proper selection of base values is fundamental to the usefulness of a per-unit system. Although per-unit quantities have been used for many years by designers of rotating machinery, the base quantities were not standardized, and it was necessary to define them in each publication. A working group of the Rotating Machinery Committee was created to propose a set of standard definitions. In 1961, a proposed standard (AIEE No. 86) was published for trial use. A critical review of that proposed standard resulted in ANSI/IEEE Std 86-1975. Recently, systems analysts and some designers of rotating electrical machinery have found it useful to have a single generic model for all types of machines. Therefore, alternate per-unit definitions, using input voltamperes as base power for induction motors, have been included in this new revision, as well as an example of an induction generator.

The members of the working group responsible for this revision were as follows:

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IEEE Recommended Practice: Definitions of Basic Per-Unit Quantities for AC Rotating Machines

1. Scope and Introduction

Since rotating electrical machines convert mechanical energy to electrical energy, or electrical energy to mechanical energy, base values for each kind of energy appear desirable in formulating a per-unit system. However, the relationship between these energies involves power factor and efficiency, which are not fundamental quantities and which vary considerably between different designs.

Traditionally, designers of generators considered rated apparent output power as base power, and designers of synchronous motors considered rated apparent input power as base power. Designers of synchronous motors used rated apparent input power as the base even though it was necessary to estimate efficiency. This permitted the use of the same design equations for both synchronous generators and synchronous motors.

Designers of induction motors generally have used rated output power as base apparent power, eliminating assumptions of power factor and efficiency. This was particularly desirable when working with small machines or machines with many poles, where accurate estimates of power factor and efficiency were difficult to make. However, some induction machines are used as induction generators, so that the definitions of input power and output power depend on the mode of operation.

Systems analysts, and some authors and machine designers, prefer the use of a single base system for all types of machines in order to simplify analysis, particularly when incorporating different machine types in a given study.

Machine equivalent circuit calculations of motors made with rated output power as a base give per-unit torque values in relatively "conventional numbers," that is, close to unity for

rated load. However, values of per-unit current are "unconventional"—rated current is not unity. When rated apparent input power is used as a base, values of per-unit current are "conventional," but torque values become "unconventional." Both systems have been in use for a long time, with proponents of each system convinced that their system best serves their purposes. Hence this recommended practice considers both bases in formulating acceptable definitions.

Since there are two systems in general use, it is important that in the transmittal of machine data, or in technical papers, the base be identified.

2. Definitions

base apparent power (alternating-current [ac] rotating machinery). A reference value expressing an electrical power rating of the machine. *Note:* Base apparent power may be either input or output power, and the numerical value may be either real power (watts [W]) or total apparent electrical power (voltamperes [VA]), depending upon machine type. Base apparent power is usually expressed in voltamperes, but any consistent set of units may be used.

For synchronous generators, induction generators, and synchronous motors, base apparent power is the total apparent electrical power at rated voltage and rated current.

In induction motors (preferred method), base apparent power is numerically equal to the rated power output.

For induction motors (alternate method), base apparent power is the total apparent electrical power at rated voltage and rated current. *Note:* When the alternate method is used it should be identified as "input voltampere base."

base voltage (ac rotating machinery). The rated phase voltage. *Note:* The value of the base voltage is the value of the rated line voltage for a delta-connected machine, and is the value of the rated line voltage divided by $\sqrt{3}$ for a wye-connected machine. Base voltage is usually expressed in volts (V), but any consistent set of units may be used.

base current (ac rotating machinery). The value of phase current corresponding to the value of base apparent power, base voltage, and the number of phases. *Note:* Base current is usually expressed in amperes (A), but any consistent set of units may be used. Base current equals the base apparent power divided by the product of base voltage and the number of phases.

base impedance (ac rotating machinery). The value of impedance corresponding to the value of the base voltage divided by the value of the base current. *Note:* Base impedance is usually expressed in ohms (Ω), but any consistent set of units may be used.

base speed (ac rotating machinery). The rated synchronous speed. *Note:* Synchronous speed equals 120 times the value of line frequency, divided by the number of poles. Base speed is usually expressed in revolutions per minute (r/min), but any consistent set of units may be used.

base torque (ac rotating machinery). The value of torque corresponding to the value of base apparent power and base synchronous

speed. The value of base torque in pound-force feet (lbf·ft) is 7.043 times the value of the base apparent power (in voltamperes [VA]), divided by the value of base speed in revolutions per minute (r/min). The value of base torque in newton meters per radian (N·m/rad) is 9.549 times the value of the base apparent power (in voltamperes), divided by the value of the base speed in revolutions per minute. *Note:* Base torque has conventionally been expressed in pound-force feet or in newton meters (N·m). To avoid confusion with the unit of energy, which is also the newton meter, the designation newton meter per radian is recommended.

per-unit value (ac rotating machinery). The actual value divided by the value of the base quantity when both actual and base values are expressed in the same units.

3. Conversions

Per-unit values in reference to a given base can be converted to per-unit values in reference to a new base by a two-step process. First, convert the per-unit value to an actual value by multiplying by the old base value and by any conversion factor necessary to make the units consistent with the new base. Second, divide this actual value by the new base value.

Appendixes

(These Appendixes are not a part of IEEE Std 86-1987, IEEE Recommended Practice: Definitions of Basic Per-Unit Quantities for Alternating-Current Rotating Machines, but are included for information only.)

**Appendix A
Application Examples**

Table A1 shows calculations for the following six examples:

- (1) An induction motor; 2 pole, 5 hp, 3460 r/min, 460 V, 3 phase, 60 Hz, 6.20 A, wye connection
- (2) An induction motor; 2 pole, 5 hp, 3460 r/min, 460 V, 3 phase, 60 Hz, 6.20 A, delta connection
- (3) An induction motor; 2 pole, 5 hp, 3460 r/min, 460 V, 3 phase, 60 Hz, 6.20 A, wye connection
- (4) A synchronous motor; 2 pole, 5 hp, 3600 r/min, 460 V, 3 phase, 60 Hz, 0.75 power factor, 7.84 A, wye connection
- (5) A synchronous generator; 2 pole, 6.25 kVA, 3600 r/min, 460 V, 3 phase, 60 Hz, 0.80 power factor, 7.84 A, wye connection
- (6) An induction generator; 2 pole, 4.25 kW, 3730 r/min, 460 V, 3 phase, 60 Hz, 6.20 A, wye connection

**Table A1
Examples of Base Values**

Base Quantity	Unit	Example (1) Induction Motor*	Example (2) Induction Motor*	Example (3) Induction Motor*	Example (4) Synchronous Motor	Example (5) Synchronous Generator	Example (6) Induction Generator
Apparent power	voltampere (VA)	$\frac{5.746 \text{ W}}{3.265.6} = 4.68$ $\frac{5.746 \text{ W}}{3.265.6} = 4.68$	$\frac{5.746 \text{ W}}{3.460} = 2.70$ $\frac{5.746 \text{ W}}{3.460} = 2.70$	$\frac{\sqrt{3} \cdot 460 \cdot 6.20}{\sqrt{3}} = 4940$ $\frac{\sqrt{3} \cdot 460 \cdot 6.20}{\sqrt{3}} = 4940$	$\frac{\sqrt{3} \cdot 460 \cdot 7.84}{3 \cdot 265.6} = 7.84$ $\frac{\sqrt{3} \cdot 460 \cdot 7.84}{3 \cdot 265.6} = 7.84$	$\frac{6.25 \cdot 1000}{\sqrt{3}} = 6250$ $\frac{6.25 \cdot 1000}{\sqrt{3}} = 6250$	$\frac{\sqrt{3} \cdot 460 \cdot 6.20}{\sqrt{3}} = 4940$ $\frac{\sqrt{3} \cdot 460 \cdot 6.20}{\sqrt{3}} = 4940$
Voltage	volt (V)	$\frac{460}{\sqrt{3}} = 265.6$ $\frac{460}{\sqrt{3}} = 265.6$	460	$\frac{460}{\sqrt{3}} = 265.6$ $\frac{460}{\sqrt{3}} = 265.6$	$\frac{460}{\sqrt{3}} = 265.6$ $\frac{460}{\sqrt{3}} = 265.6$	$\frac{460}{\sqrt{3}} = 265.6$ $\frac{460}{\sqrt{3}} = 265.6$	$\frac{460}{\sqrt{3}} = 265.6$ $\frac{460}{\sqrt{3}} = 265.6$
Current	ampere (A)	$\frac{3730}{3.265.6} = 4.68$ $\frac{3730}{3.265.6} = 4.68$	$\frac{3730}{3.460} = 2.70$ $\frac{3730}{3.460} = 2.70$	$\frac{4940}{3 \cdot 265.6} = 6.20$ $\frac{4940}{3 \cdot 265.6} = 6.20$	$\frac{6250}{3 \cdot 265.6} = 7.84$ $\frac{6250}{3 \cdot 265.6} = 7.84$	$\frac{6250}{3 \cdot 265.6} = 7.84$ $\frac{6250}{3 \cdot 265.6} = 7.84$	$\frac{4940}{3 \cdot 265.6} = 6.20$ $\frac{4940}{3 \cdot 265.6} = 6.20$
Impedance	ohm (Ω)	$\frac{265.6}{4.68} = 56.75$ $\frac{265.6}{4.68} = 56.75$	$\frac{460}{2.70} = 170.4$ $\frac{460}{2.70} = 170.4$	$\frac{265.6}{6.20} = 42.84$ $\frac{265.6}{6.20} = 42.84$	$\frac{265.6}{7.84} = 33.88$ $\frac{265.6}{7.84} = 33.88$	$\frac{265.6}{7.84} = 33.88$ $\frac{265.6}{7.84} = 33.88$	$\frac{265.6}{6.20} = 42.84$ $\frac{265.6}{6.20} = 42.84$
Speed	revolutions per minute (r/min)	$\frac{120 \cdot 60}{2} = 3600$ $\frac{120 \cdot 60}{2} = 3600$	$\frac{120 \cdot 60}{2} = 3600$ $\frac{120 \cdot 60}{2} = 3600$	$\frac{120 \cdot 60}{2} = 3600$ $\frac{120 \cdot 60}{2} = 3600$	$\frac{120 \cdot 60}{2} = 3600$ $\frac{120 \cdot 60}{2} = 3600$	$\frac{120 \cdot 60}{2} = 3600$ $\frac{120 \cdot 60}{2} = 3600$	$\frac{120 \cdot 60}{2} = 3600$ $\frac{120 \cdot 60}{2} = 3600$
Torque	pound-force foot (lbf. ft)	$\frac{7.043 \cdot 3730}{3600} = 7.30$ $\frac{7.043 \cdot 3730}{3600} = 7.30$	$\frac{7.043 \cdot 3730}{3600} = 7.30$ $\frac{7.043 \cdot 3730}{3600} = 7.30$	$\frac{7.043 \cdot 4940}{3600} = 9.66$ $\frac{7.043 \cdot 4940}{3600} = 9.66$	$\frac{7.043 \cdot 6250}{3600} = 12.23$ $\frac{7.043 \cdot 6250}{3600} = 12.23$	$\frac{7.043 \cdot 6250}{3600} = 12.23$ $\frac{7.043 \cdot 6250}{3600} = 12.23$	$\frac{7.043 \cdot 4940}{3600} = 9.66$ $\frac{7.043 \cdot 4940}{3600} = 9.66$
Torque	newton meter per radian (N.m/rad)	$\frac{9.549 \cdot 3730}{3600} = 9.89$ $\frac{9.549 \cdot 3730}{3600} = 9.89$	$\frac{9.549 \cdot 3730}{3600} = 9.89$ $\frac{9.549 \cdot 3730}{3600} = 9.89$	$\frac{9.549 \cdot 4940}{3600} = 13.10$ $\frac{9.549 \cdot 4940}{3600} = 13.10$	$\frac{9.549 \cdot 6250}{3600} = 16.58$ $\frac{9.549 \cdot 6250}{3600} = 16.58$	$\frac{9.549 \cdot 6250}{3600} = 16.58$ $\frac{9.549 \cdot 6250}{3600} = 16.58$	$\frac{9.549 \cdot 4940}{3600} = 13.10$ $\frac{9.549 \cdot 4940}{3600} = 13.10$

* Examples (1) and (2) show the preferred system for induction motors; example (3) shows the alternate system based on voltampere (VA) input. For comparison, the induction machines in examples (1), (3), and (6) are the same physical machine, operated in (1) and (3) as a motor and in (6) as an induction generator.

Appendix B
Examples of Machine Constants

Ratings for the six examples in Table B1 are the same as those given in Appendix A.

Table B1
Examples of Machine Parameters

Parameter	Unit	Example (1) Induction Motor	Example (2) Induction Motor	Example (3) Induction Motor	Example (4) Synchronous Motor	Example (5) Synchronous Generator	Example (6) Induction Generator
Resistance							
Stator, R_1	ohm (Ω)	2.23	6.69	2.23	2.26	2.26	2.23
Rotor, R_2	ohm	1.75	5.25	1.75	—	—	1.75
Direct axis, X_d	ohm	—	—	—	36.72	36.72	—
Quad. axis, X_q	ohm	—	—	—	12.68	12.68	—
Exciting, X_m	ohm	111.9	335.8	111.9	—	—	111.9
Leakage, X	ohm	5.25	15.75	5.25	—	—	5.25
Total loss	watt (W)	812	812	812	957	957	836
Phase current	ampere (A)	6.20	3.58	6.20	7.84	7.84	6.20
Output power	watt	3730	3730	3730	3730	5000	4250
Torque							
	pound-force foot (lbf.ft)	7.59	7.59	7.59	7.29	11.65	9.60
	newton meter per radian (N·m/rad)	10.29	10.29	10.29	9.88	15.80	13.02

NOTES:

- (1) Typical numerical values of resistance and reactance have been selected for the examples. For comparison, the synchronous motor and the synchronous generator are the same machine operated in the two modes.
- (2) For comparison, the induction machines in examples (1), (3), and (6) are the same machine, operated in (1) and (3) as a motor, and in (6) as an induction generator.
- (3) All resistance and reactance values are phase values and refer to the stator.

Appendix C
Calculations of Per-Unit Values

Ratings for the six examples in Table C1 are the same as those given in Appendix A.

Table C1
Calculation of Per-Unit Values

Per Unit	Example (1) Induction Motor	Example (2) Induction Motor	Example (3) Induction Motor	Example (4) Synchronous Motor	Example (5) Synchronous Generator	Example (6) Induction Generator
Resistance	$R_1 = \frac{2.23}{56.75} = 0.039$ $R_2 = \frac{1.75}{56.75} = 0.031$	$R_1 = \frac{6.69}{170.4} = 0.039$ $R_2 = \frac{5.25}{170.4} = 0.031$	$R_1 = \frac{2.23}{42.84} = 0.052$ $R_2 = \frac{1.75}{42.84} = 0.041$	$R = \frac{2.26}{33.88} = 0.067$	$R = \frac{2.26}{33.88} = 0.067$	$R_1 = \frac{2.23}{42.84} = 0.052$ $R_2 = \frac{1.75}{42.84} = 0.041$
Reactance	$X_m = \frac{111.9}{56.75} = 1.97$ $X = \frac{5.25}{56.75} = 0.092$	$X_m = \frac{335.8}{170.4} = 1.97$ $X = \frac{15.75}{170.4} = 0.092$	$X_m = \frac{111.9}{42.84} = 2.61$ $X = \frac{5.25}{42.84} = 0.123$	$X_d = \frac{36.72}{33.88} = 1.08$ $X_q = \frac{12.68}{33.88} = 0.374$	$X_d = \frac{36.72}{33.88} = 1.08$ $X_q = \frac{12.68}{33.88} = 0.374$	$X_m = \frac{111.9}{42.84} = 2.61$ $X = \frac{5.25}{42.84} = 0.123$
Rated voltage	$\frac{460/\sqrt{3}}{265.6} = 1.00$	$\frac{460}{460} = 1.00$	$\frac{460/\sqrt{3}}{265.6} = 1.00$	$\frac{460/\sqrt{3}}{265.6} = 1.00$	$\frac{460/\sqrt{3}}{265.6} = 1.00$	$\frac{460/\sqrt{3}}{265.6} = 1.00$
Rated current	$\frac{6.20}{4.68} = 1.33$	$\frac{358}{2.70} = 1.33$	$\frac{6.20}{6.20} = 1.00$	$\frac{7.84}{7.84} = 1.00$	$\frac{7.84}{7.84} = 1.00$	$\frac{6.20}{6.20} = 1.00$
Rated output power	$\frac{3730}{3730} = 1.000$	$\frac{3730}{3730} = 1.000$	$\frac{3730}{4940} = 0.755$	$\frac{3730}{6250} = 0.597$	$\frac{5000}{6250} = 0.800$	$\frac{4250}{4940} = 0.860$
Loss*	$\frac{812}{3730} = 0.218$	$\frac{812}{3730} = 0.218$	$\frac{812}{4940} = 0.164$	$\frac{957}{6250} = 0.153$	$\frac{957}{6250} = 0.153$	$\frac{836}{4940} = 0.169$
Input power*	$1.000 + 0.218 = 1.218$	$1.000 + 0.218 = 1.218$	$0.755 + 0.164 = 0.919$	$0.597 + 0.153 = 0.750$	$0.800 + 0.153 = 0.953$	$0.860 + 0.169 = 1.029$
Torque (lbf-ft base)	$\frac{7.59}{7.30} = 1.040$	$\frac{7.59}{7.30} = 1.040$	$\frac{7.59}{9.66} = 0.786$	$\frac{7.29}{12.23} = 0.596$	$\frac{11.65}{12.23} = 0.953$	$\frac{9.60}{9.66} = 0.994$
Torque (N·m/rad base)	$\frac{10.29}{9.89} = 1.040$	$\frac{10.29}{9.89} = 1.040$	$\frac{10.29}{13.10} = 0.786$	$\frac{9.88}{16.58} = 0.596$	$\frac{15.80}{16.58} = 0.953$	$\frac{13.02}{13.10} = 0.994$
Rated speed	$\frac{3460}{3600} = 0.961$	$\frac{3460}{3600} = 0.961$	$\frac{3460}{3600} = 0.961$	$\frac{3600}{3600} = 1.000$	$\frac{3600}{3600} = 1.000$	$\frac{3730}{3600} = 1.036$

* At rated load.

